THE ASTROPHYSICAL JOURNAL, 267:863–871, 1983 April 15

A THERMAL MODEL OF SUNSPOT INFLUENCE ON SOLAR LUMINOSITY

P. FOUKAL, L. A. FOWLER, AND M. LIVSHITS
Atmospheric and Environmental Research, Inc., Cambridge, Mass.
Received 1982 August 16; accepted 1982 October 12

ABSTRACT

Recent measurements of the solar irradiance have confirmed that sunspots block energy flow to the photosphere in rough proportion to their area and photometric contrast. We have constructed a time-dependent two-dimensional model of heat flow blocking in a turbulent convective layer, to investigate the physical interpretation of the observed irradiance dips. Our numerical model shows how formation of a spot at or below the photosphere leads to heating of surrounding convective layers over a diffusive time scale $\tau_D \sim 10^4$ s. This heating rapidly propagates outward, storing the blocked heat throughout the convection zone. The stored thermal (and potential) energy is only released very slowly by radiation through a gradually increasing photospheric temperature. The very long radiative time scale $\tau_R > 10^{10}$ s for this release is quite insensitive to reasonable uncertainties in the model parameters or the diffusion approximation we have used. We point out that this very efficient storage implies a sunspot contribution to the modulation of $L_{\odot}$ over the 11 year cycle, at a level somewhat below 0.1%. Our study indicates that the amplitude, duration, shape, and phase of the observed spot-correlated irradiance dips can be easily explained by extending a conventional thermal blocking model of spots to include time dependence. We find no reason to expect that the contribution of faculae to S and $L_{\odot}$ cancels that of spots, on any time scale.

Subject headings: radiative transfer — Sun: activity — Sun: general — Sun: sunspots

I. INTRODUCTION

Radiometers on the *Solar Maximum Mission* and *Nimbus 7* satellites show clear depressions in the solar irradiance S, whose timing correlates well with disk passage of large sunspot groups (Willson *et al.* 1981; Hickey *et al.* 1980). The amplitude (0.2%–0.4%), duration (10–20 days), and shape of many of the larger dips agree well with the irradiance decrease calculated (Foukal, Mack, and Vernazza 1977; Willson *et al.* 1981), on the assumption that spots block photospheric heat flow in proportion to their area and bolometric contrast.

The irradiance dips are too large to be compensated by observed ultraviolet flux variations below 0.18 μm (e.g., Heath 1980; Heath and Thekaekara 1977), to which the radiometers may be less sensitive. Nevertheless, the dips do not directly require changes in solar luminosity, $L_{\odot}$. For instance, the missing flux might emerge at a large angle to the radiometer's line of sight, in the anisotropic radiation field of bright magnetic faculae (e.g., Chapman 1980; Oster, Schatten, and Sofia 1982). But large facular areas often occur without detectable sunspots on the disk, and their lifetimes are also much longer (e.g., Kiepenheuer 1953). Detailed balance of radiative fluxes between spots and faculae would then require efficient transfer and storage of roughly 10^{36} ergs over months between these magnetic features. A physically convincing mechanism to achieve this detailed balance has yet to be put forward.

We show in this paper that the observed properties of the irradiance dips can be easily explained by the conventional thermal blocking model of sunspots (Biermann 1941; Cowling 1976) extended to include time dependence. Our model does not rule out energy transfer between spots and faculae, but it does not require it. Our analysis in §§ II–III indicates that the heat blocked (in proportion to a spot's area and contrast) is stored very efficiently in the slightly increased thermal (and potential) energy of the solar convection zone. The radiative flux blocked during high sunspot activity periods is only radiated away over many subsequent 11 year cycles. In § IV we point out that this efficient storage implies a contribution to variation of $L_{\odot}$ and S over the 11 year cycle, at an amplitude that can be computed from the known variation of sunspot areas.

II. A TIME-DEPENDENT THERMAL SUNSPOT MODEL

a) *Physical Assumptions*

Recent work on thermal obstacles in the solar convection zone (e.g., Spruit 1977; Clark 1979) shows that the most restrictive photometric features of a spot, namely the sharpness of the umbral edge and faintness of any photospheric bright ring around the penumbra, can be explained in terms of a relatively simple diffusion approximation to heat flow around spots. In such models (see also Parker 1974 and Eschrich and Krause 1977), heat transport is represented as turbulent eddy diffusion, characterized by an eddy heat conductivity K. Within the spot, convection is taken to be blocked by a strong vertical magnetic field, as first proposed by Biermann (1941) (see also Cowling 1976), and the value of K is taken to be small.

The diffusion approximation requires that the scales of convective heat transport near the spot be small relative to the dimensions of the spot itself. Granular convection in a shallow layer near the photosphere satisfies this requirement. But the scales of convection that carry heat at greater depths are quite unknown. Supergranules of dimensions between 15 and 30,000 km are a candidate (e.g., Simon and Weiss 1968). But the finding that they do not show any detectable temperature gradient between cell centers and edges (Beckers 1968; Worden 1975) casts doubt on their heat transport efficiency. Recent studies of line shifts in supergranulation (Giovanelli 1980; Miller, Foukal, and Keil 1982) also remove previous evidence for vertical motions. These observations do not preclude substantial heat transport by supergranules at greater depth. But neither do observations rule out heat transport primarily by small turbulent eddies throughout the region affected by the spot, indeed throughout the solar convection zone (Durney 1976; Durney and Spruit 1979).

The diffusion model does not explicitly deal with the dynamics of the spot magnetic field; it merely represents the blocking of transverse convective motions (by the Lorentz force $\boldsymbol{J} \times \boldsymbol{B}$) as a local increase in thermal impedance, or a thermal plug. Thus, the model does not consider the work done in concentrating the magnetic field of the spot in the first place. But it is easy to show that even under the most favorable circumstances, that work must be small compared to the 10^{36} ergs that would need to be stored, if the observed irradiance dips were to be explained as a consequence of rapid conversion of convective heat flux into magnetic field energy (Foukal 1981).

Given the success of the steady heat flow models in explaining the temperature distribution around spots, it seems worthwhile to inquire into their time-dependent properties. Our aim is to understand how changes in spot area and depth might lead to heat storage through modification of the convection zone thermal structure outside the spot.

b) Equation of Heat Flow and Initial and Boundary Conditions

Following the usual notation (e.g., Cox and Giuli 1968) we write the diffusion equation in terms of the superadiabatic temperature gradient $\nabla T - \nabla T_{\rm ad}$ as:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[K(\nabla T - \nabla T_{\rm ad})\right]. \qquad (1)$$

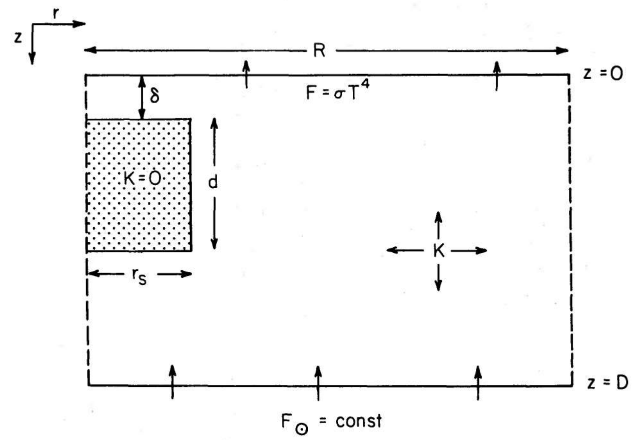

FIG. 1.—Schematic diagram illustrating the geometry and boundary conditions of the two-dimensional thermal sunspot model. The model is symmetric about the z-axis.

In our plane-parallel, cylindrically symmetric model (illustrated schematically in Fig. 1), z and r are, respectively, the depth and radius coordinates. The convection zone is taken to have depth $z = D$ and radius $r = R$. The sunspot of radius $r = r_s$ and depth $z = d$ is a vertical cylinder placed along the z-axis. Its upper surface may be located at the photosphere ($z = 0$), or at some depth $z = \delta$.

The initial ($t < 0$) depth profiles of ∇T, $\nabla T_{\rm ad}$, density ρ, and specific heat C_p in the convection zone outside the spot, are taken from the convection zone model of Baker and Temesvary (1966). We calculate an initial profile of $K(z)$ (including transport by both turbulent gas motions and photon diffusion) from

$$K(z) = F_\odot/(\nabla T - \nabla T_{\rm ad}), \qquad (2)$$

where $F_\odot = 6.207 \times 10^{10}$ ergs cm^{-2} s^{-1} is the undisturbed radiative flux density from the photosphere. Since the actual (tensor) form of K is highly uncertain (e.g., Unno 1961; Spruit 1977), we assume here that K is isotropic in the convecting region outside the spot. Inside the spot, we set $K = 0$.

The upper boundary condition on the model; $F = \sigma T_{\rm phot}^4$ expresses blackbody radiation at an effective temperature $T_{\rm phot} = 5750$ K. The lower boundary condition is $F = F_\odot$ (constant). This assumes no effect of the spot on thermonuclear energy generation or on energy storage below the convection zone. Finally, we take $F = 0$ across the lateral boundary of the convection zone ($r = R$), and also $F = 0$ across all boundaries of the spot itself.

Numerical integrations of equation (1) were carried out using a two-level explicit finite difference scheme (Richtmeyer and Morton 1967). Integrations were started at $t = 0$, when a spot is switched on in the initially steady Baker and Temesvary convection zone. The model then computed the perturbed superadiabatic temperature gradient field ($\nabla T - \nabla T_{ad}$), holding $\rho(z)$, $C_p(z)$, and $K(z)$ constant in time. To simulate the eventual disappearance of the spot at $t = t_s$, the procedure could also be reversed by using the temperature field at $t = t_s$ as an initial condition for an integration in which the spot had been removed.

The model calculates the spatially integrated flux $\Phi = \int_{\pi R^2} F ds$ through the photosphere as a function of time, showing the spot's effect on the luminosity. Isotherms of ($T - T_{ad}$) are plotted to investigate the shape of the heat flow perturbation caused by the spot. The radial flux distribution $F(r)$ was also computed to allow comparison between the brightness distribution predicted by our model, and that observed photometrically (e.g., Fowler, Foukal, and Duvall 1982).

III. RESULTS OF MODEL INTEGRATIONS AND THEIR INTERPRETATION

a) The Diffusion Time Scale, t_D

Figure 2 shows the isotherm perturbations caused by a spot of $r_s = d = 10^4$ km and $\delta = 0$ in a convecting layer with $D = R = 5 \times 10^4$ km. This time sequence illustrates how insertion of the spot rapidly changes the equilibrium isotherms at $t < 0$ to a new heat flow pattern near the spot, within 8 hr. The change of isotherm shape shows that the convection zone under the spot heats up immediately, and this heating rapidly propagates outward from the spot and down into the convection zone. This heating represents storage of the blocked heat in the thermal energy of the convection zone outside the spot.

The time required to establish the new pattern of quasi-steady isotherms over a region comparable to the spot's dimensions is (Fig. 2) roughly 5×10^4 s. This is in good agreement with the diffusion time scale $\tau_D = L^2/\lambda$, where $\lambda = K/\rho C_p$ is the eddy heat diffusivity with L taken as the doubling scale ($\sim 10^4$ km) of the sunspot's perturbations to ($\nabla T - \nabla T_{ad}$), calculated in the model.

b) Propagation of the Spot's Thermal Signal, and the Depth of Heat Storage

Figure 3 shows a time series of isotherms of ($T - T_{ad}$) illustrating the propagation of the spot's thermal perturbations through a convection zone of $D = R = 10^5$ km, for the two cases of a shallow ($d = 3300$ km, $r_s = 10^4$ km) and deep ($d = r_s = 10^4$ km) spot. This time series shows that a heating of roughly 1 K reaches a depth $z = 3 \times 10^4$ km, and a horizontal distance $r \approx 2.5 \times 10^4$ km within $t = 25$ days. The shape of the isotherm perturbation near the shallow and deep spots differs significantly (see Fig. 3), but the dimension of the convection zone region affected by the perturbation after 25 days, is closely similar. This shows that the propagation time of the spot's thermal signal through the convection zone is not very sensitive to the uncertain sunspot depth, d.

Although the temperature perturbation rapidly decreases with depth, the plasma heat capacity at $z = D$ is roughly 200 times greater than near $z = 0$, so that the heat stored $\rho C_p \Delta T(z)$ at large z might rapidly become appreciable, even if $\Delta T(D) \ll \Delta T(0)$, as Figure 3 indicates. Table 1 shows the time change in the depth profile of heat storage. It can be seen that roughly 2.5% of the total blocked heat is already stored at $z \sim D$, at $t = 25$ days. This indicates that the heat blocked at the photosphere by a spot is rapidly redistributed for global storage throughout the convection zone, within time scales comparable to the spot lifetime.

c) Sunspot Bright Rings

Figure 4 shows plots of the excess relative flux $\Delta F(r)/F(\infty)$ outside the spot boundary. The two cases illustrated refer to a shallow ($d = 3300$ km, $r_s = 1 \times 10^4$ km) and deep ($d = r_s = 10^4$ km) spot in a convection zone with $D = R = 10^5$ km.

The plots show that our model predicts bright rings of amplitude below 0.3%, even for a very shallow spot. These calculated amplitudes are consistent with results of our recent photometric study of bright rings (Fowler, Foukal, and Duvall 1982). The spatial extent of the rings also illustrates that the observable brightness perturbation around the spot is confined to a relatively small region of 2–3 r_s outside the spot boundary.

d) The Thermal Storage Time

Figure 5 plots the relative deficit in total flux $\Delta\Phi(t)/\Phi$ after a spot is switched on at $t = 0$, and then subsequently switched off at $t = t_s$. Insertion of the spot (blocking all radiation from an area πr_s^2) immediately reduces Φ by the factor

$$\Delta\Phi/\Phi = -r_s^2/R^2. \tag{3}$$

If the convective layers perturbed by the spot had negligible heat capacity, the blocked heat would immediately raise the temperature of the surrounding photosphere to a new equilibrium value T'_{phot} determined by;

$$\pi\left(R^2 - r_s^2\right)\sigma T'^4_{phot} = \pi R^2 T^4_{phot}, \tag{4}$$

in which case our model would predict no effect of spots on Φ (or on $L_\odot$).

Figure 5 shows that the actual recovery of Φ toward its original value at $t < 0$ is very slow. Even for a shallow convective layer of $D = 10^4$ km, the deficit $\Delta\Phi$ given by equation (3) is only reduced by 15% after $t = 25$ days. The times t_{eq} required to achieve the new equilibrium defined by equation (4) are given in Table 2 for various

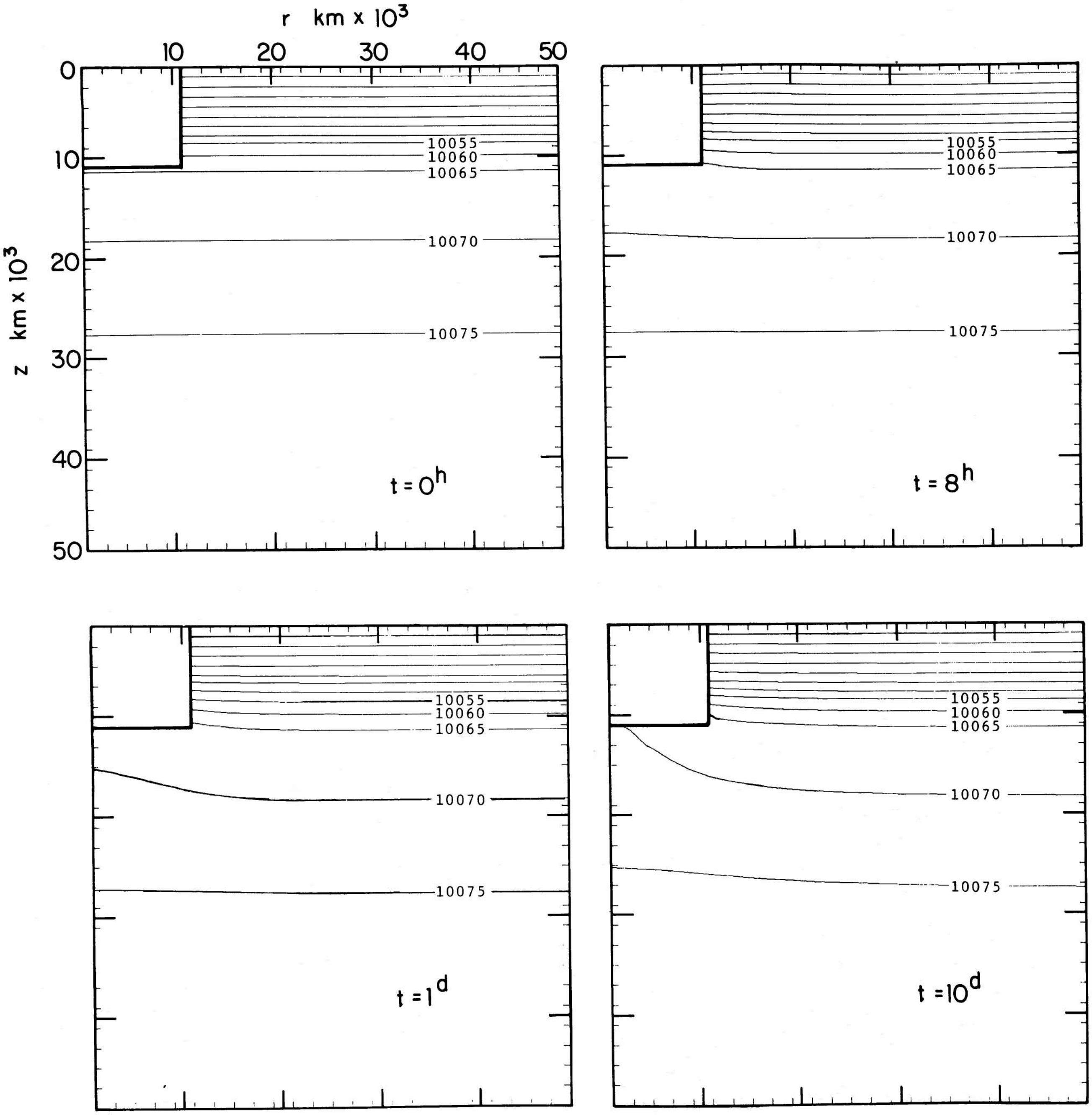

FIG. 2.—Time dependence of $(T - T_{ad})$ isotherms for a spot of $d = 10^4$, $r_s = 10^4$ km inserted into a convection zone of $D = 5 \times 10^4$ km, $R = 5 \times 10^4$ km. The isotherms are shown before the spot is inserted at $t = 0$, and at $t = 8$ h, 1 day, and 10 days.

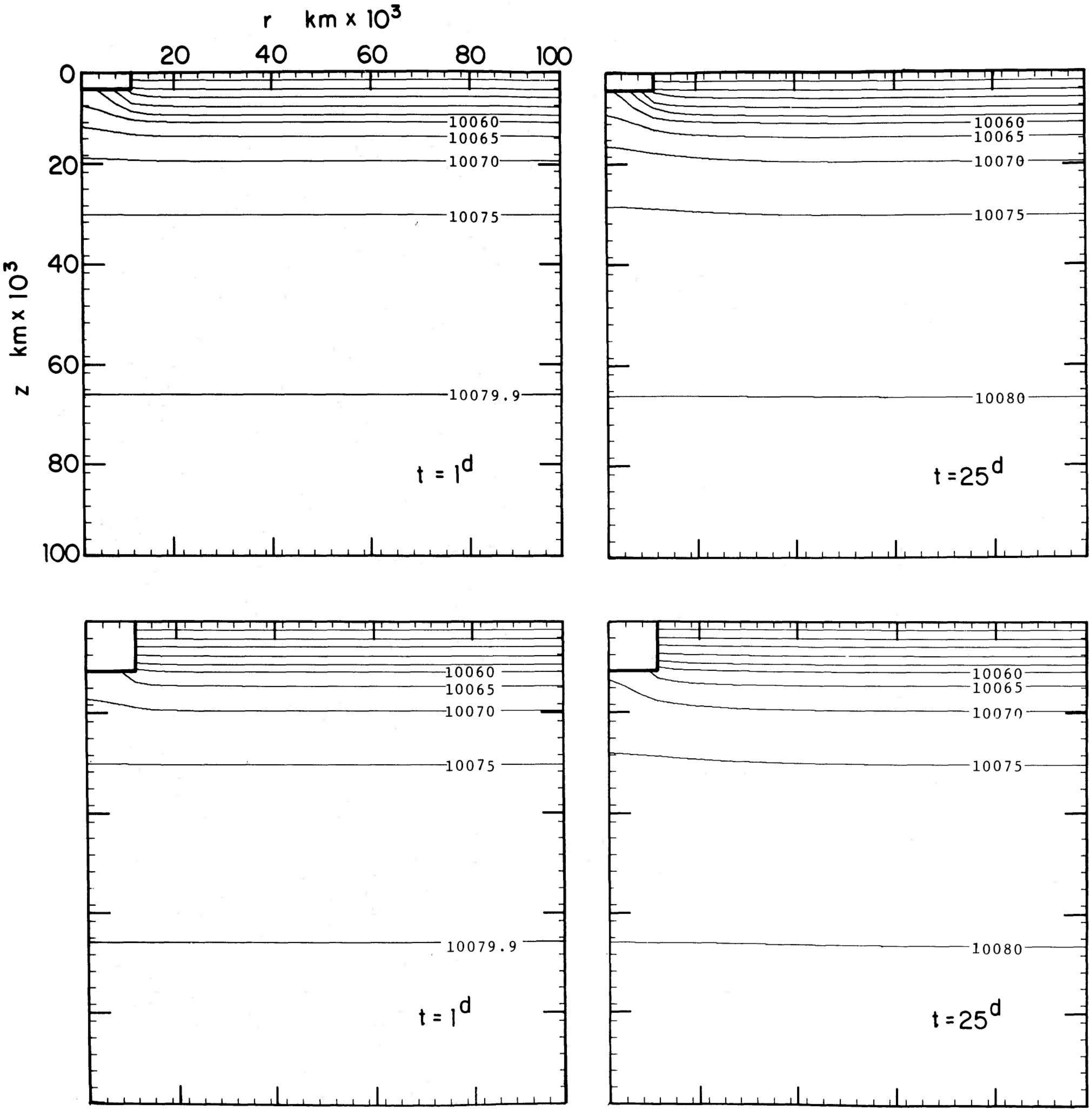

FIG. 3.—Propagation of the thermal disturbance in a convection zone of $D = R = 10^5$ km. The shallow spot shown has dimensions $d = 3.3 \times 10^3$ km, $r_s = 1 \times 10^4$ km. The deep spot has dimensions $d = r_s = 10^4$ km. The isotherms are shown in each case for $t = 1$ and 25 days. Note that the change in the lowest isotherm at $z = 6 \times 10^4$ km shows heating of 0.1 K (10,079.9 to 10,080) over 25 days.

TABLE 1

PERCENTAGE OF HEAT STORED IN A LAYER OF THICKNESS $\Delta z = 10^4$ km, CENTERED AT z[a]

	t (d)		
z (km $\times 10^4$)	5	10	25
2	51.4	44.1	37.1
4	12.4	13.9	13.4
6	2.1	4.0	5.5
8	0.3	1.2	3.0
10	0.03	0.4	2.5

[a] Results based on a model run with; $d = r_s = 10^4$ km, $D = R = 10^5$ km. Note that the percentages do not sum to 100, because values at $z = 3, 5, 7$, and 9 ($\times 10^4$ km) are not tabulated.

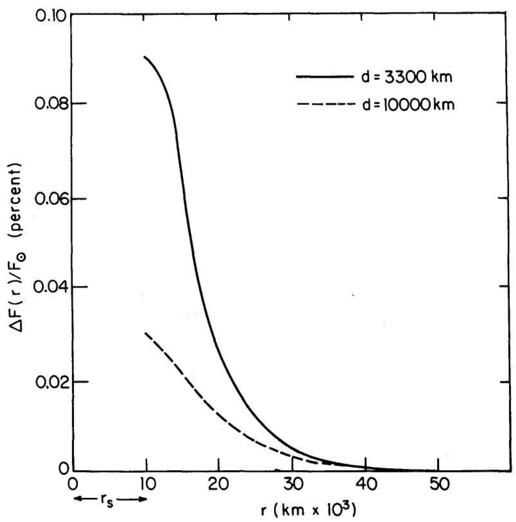

FIG. 4.—Radial profiles of the excess radiative flux $\Delta F(r)/F_{\odot}$ for two spots of depths $d = 3.3 \times 10^3$, 1×10^4 km, and $r_s = 5 \times 10^3$ km, in a convection zone with $D = R = 10^5$ km.

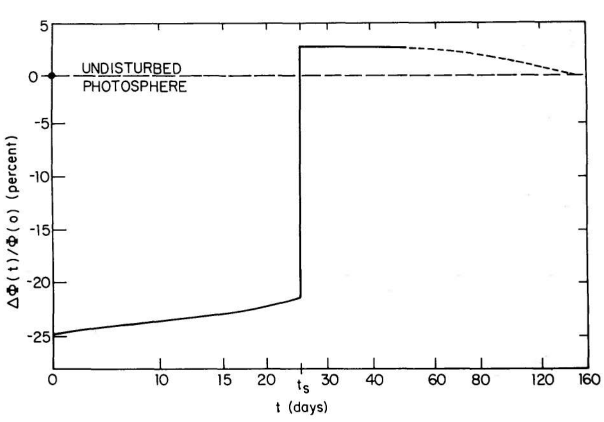

FIG. 5.—Time dependence of the total flux $\Phi(t)$ for a spot of $d = 1 \times 10^3$, $r_s = 5 \times 10^3$ in a shallow convection zone of $R = 10^4$ km, $D = 10^4$ km. The spot is removed at $t = t_s$.

convective layer depths and sunspot dimensions. Lower limits to t_{eq} are obtained by linear extrapolation of the decrease $\Delta\Phi$ obtained in a 25 day simulation, such as Figure 5. For $D = 10^4$ km, $t_{eq} \approx 195$ days. For more realistic values of D, $t_{eq} \gg 10$ yr, although the exact values are difficult to estimate with reasonable run times of the model. The main point is that over the 10–20 day duration of the observed irradiance dips, almost none of the blocked heat would be expected to emerge, so that storage is essentially perfect, and equation (3) should hold exactly. Since the values of t_{eq} in Table 2 are insensitive to changes in r_s and d, we expect that our result depends little on the unknown depths d and shape (r_s/d) of typical spots.

More generally, for n spots of photometric contrast $C_s = I_{spot}/I_{phot}$, equation (3) leads to the formula

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta S}{S} = (C_s - 1) \sum_{i=1}^{n} A_i , \qquad (5)$$

where $A_i = r_i^2/R^2$ is the fractional area of the projected photospheric disc covered by each spot. Equation (5) has been previously derived (Foukal, Mack, and Vernazza 1977; Willson *et al.* 1981; Foukal 1981) on the *assumption* that perfect storage obtains over the spot lifetime, and this relation has been shown (Willson *et al.* 1981; Foukal 1981) to fit many of the largest dips seen in the *SMM* ACRIM and *Nimbus 7* ERB irradiance data.

e) Fate of the Stored Heat after the Spot Disappears

The behavior of Φ after the spot is removed at $t = t_s$ is also shown in Figure 5. At $t = t_s$, Φ immediately rises to a value slightly in excess of its value at $t < 0$, since the radiating area of the photosphere is restored to πR^2, but the photospheric temperature has increased by a small amount ΔT_{phot}. Figure 5 shows that Φ then slowly decays for $t > t_s$; the photosphere must cool, since the convection zone is radiating energy faster than it receives heat from the solar interior. The value of this decay time t'_{eq} is also given in Table 2. It agrees approximately with the value for t_{eq} for a convection zone of $D = 10^4$ km.

The interpretation of the storage time $t'_{eq} \sim t_{eq}$ is easily visualized. The blocked heat stored over the depth l is given by $\int_l \rho(z) C_p(z) \Delta T(z)\, dz$, while the excess photospheric radiation at the slightly increased photospheric temperature is $4\sigma T^3 \Delta T_{phot}$. The equilibration time scale for radiating the stored heat is then

$$\tau_R \sim \overline{\rho C_P}\, l / 4\sigma T^3 , \qquad (6)$$

taking $\Delta T(z) \sim \Delta T_{phot}$.

Table 2 gives values of τ_R for comparison with t_{eq}, t'_{eq}. The τ_R values are calculated using $l = D$, based on the

TABLE 2

CALCULATED EQUILIBRATION AND RADIATIVE RELAXATION TIMES OF CONVECTIVE LAYERS

D (km × 10^4)	r_s (km × 10^4)	d (km × 10^4)	R (km × 10^4)	t_{eq} (s)	t'_{eq} (s)	τ_R (s)
1	1	0.2	3	$\gtrsim 1.7 \times 10^7$	$\geq 1.3 \times 10^7$	2.8×10^7
5	1	0.2	3	$\gg 3.1 \times 10^8$	...	7.6×10^9
10	1	0.3	3	$\gg 4.6 \times 10^8$	...	5.9×10^{10}

results of § IIIb, and Table 1 for the depth of energy storage. For $D = 10^4$ km, the value of τ_R is only a few times longer than the maximum run time $t = 25$ of our model, and here we find $t_{eq} \sim \tau_R$. For $D = 5 \times 10^4$ km and $D = 1 \times 10^5$ km, $\tau_R \sim 10^2$–10^3 yr. This is consistent with our finding above that $t_{eq} \gg 10$ yr, although exact correspondence cannot be checked with a reasonable run time since Φ recovers so very slowly. However, we can infer that the storage time for heat blocked by a spot is reasonably estimated by equation (6), using $l \sim D$.

f) Submerged Flux Tubes and the Relative Phase of Changes in $L_\odot$ and A_i

Figure 6 plots the relative deficit $\Delta\Phi(t)/\Phi(0)$ for a thermal obstruction submerged at a depth $\delta = 6 \times 10^4$ km, with $d = 1 \times 10^4$ km, $r_s = 4 \times 10^4$ km, and $R = 2D = 2 \times 10^5$ km. The main feature of interest in Figure 6 is the gradual decrease of Φ at $t > 0$. Isotherm plots show that this gradual decrease over a time scale $\Delta T \sim 30$ days results from the gradual propagation of cooling above the obstacle to the photosphere. Integrations with shallower layers (Table 3) show that after this "thermal shadow" has arrived at the photosphere, Φ begins to slowly recover toward its value at $t < 0$, over the long time scale t_{eq}.

Table 3 also shows the temperature amplitude ΔT of the thermal shadow at the photosphere, for several values of d, r_s, and δ as well as D and R. As noted previously by Spruit (1977) from steady heat flow calculations, ΔT rapidly decreases as δ/r_s increases. The detectable limit $\Delta T \sim 10$ K (Fowler, Foukal, and Duvall 1982) occurs at $\delta \sim 2r_s$. But this result is quite sensitive to K, and photometric investigation of such shadows is of considerable interest in placing constraints on the actual value of K to be used in the convection zone models.

Thermal shadows are only to be expected if the mean rate of rise of magnetic flux tubes is less than the mean vertical propagation rate derived in § IIIb for the thermal signal. In this event, we also expect to find that dips in $L_\odot$ will lead increases of spot area at the photosphere. Cross-correlations between the time series of daily irradiances and sunspot areas suggest that such a phase advance of $\Delta t \sim 1$–2 days (Foukal and Vernazza 1979; Foukal 1981), but the error bars are still large. The rate of flux tube rise in these layers estimated by observations (e.g., Frazier 1972) and theory (e.g., Parker 1975) is also uncertain. Photometric investigations of thermal shadows could yield valuable constraints on these important quantities.

As regards the predicted effect of submerged magnetic obstacles on the luminosity, we see from Figure 6 that the relative deficit $\Delta\Phi/\Phi \simeq 1 \times 10^4$ for an obstacle of relative area $r_s^2/R^2 \sim 4\%$, submerged at $\delta \sim r_s$. We might expect then that a long-lived submerged active complex covering 10% or more of the solar disc could produce a rotational modulation of $L_\odot$ at a radiometrically detectable level.

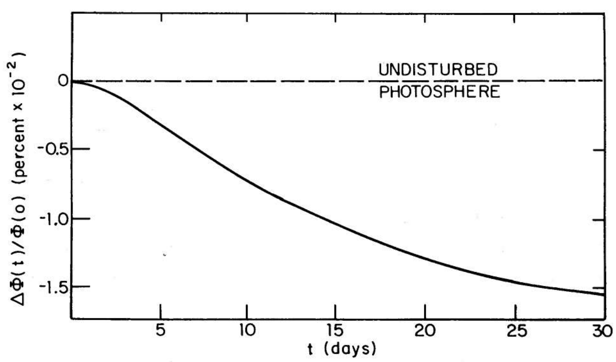

FIG. 6.—Time dependence of total flux $\Phi(t)$ for a submerged spot; $\delta = 6 \times 10^4$ km, $d = 1 \times 10^4$ km, $r_s = 4 \times 10^4$ km. The convection zone dimensions are $D = 1 \times 10^5$ km, $R = 2 \times 10^5$ km.

TABLE 3

CHARACTERISTICS OF THERMAL SHADOWS

Sunspot Dimensions (km × 10^4)			Convection Zone Dimensions (km × 10^4)			
δ	d	r_s	D	R	Δt[a] (d)	ΔT (K)
0.2	0.5	0.5	1.5	1.5	3	714
0.5	0.5	0.5	1.5	1.5	3	55
0.5	0.16	0.5	5	5	3	70
1	0.16	0.5	5	5	3	6
3	1	4	10	20	50	32
6	1	4	10	20	60	2

[a]Approximate time needed for full temperature difference ΔT to appear at the photosphere.

IV. VARIATIONS OF S OVER THE 11 YEAR CYCLE

The highly effective heat storage over time scales given roughly by τ_R, implies that heat blocked by spots near the peak of the activity cycle will only be slowly released over many subsequent 11 year cycles. As noted above, under such conditions of excellent storage, the time behavior of the irradiance deficit caused by spots can be calculated from equation (5). It follows that the value of S will be depressed at solar activity maximum, when ΣA_i is large. When the spots decay, the solar luminosity and irradiance will return to a value only negligibly higher than its value before their formation. The elevated luminosity will decay slowly over ensuing millenia, as the stored heat is radiated away. This long storage time also implies that the Sun's present luminosity depends on mean solar activity levels over past centuries.

Our model thus provides some physical justification for calculating the time history of S over the time scales of decades, from the tabulated areas of sunspots (Hoyt 1978; Eddy, Hoyt, and White 1982). Such computations indicate a variation in the annual mean of S, of typically less than 0.1%, between sunspot activity maximum and minimum.

V. DISCUSSION AND CONCLUSIONS

The main result of this study (see also Foukal 1981 for a discussion of early results from this model) is the finding that the blocking of heat flow by magnetic spots can lead to very efficient storage of this energy as a small increase in the internal energy of the solar convection zone. This finding increases our confidence in the interpretation of the observed irradiance dips as true decreases of solar luminosity.

This highly efficient storage over at least centuries does not follow from dimensional analysis of the heat diffusion equation (1), which leads to the much shorter time scale of days, associated with diffusion of the spot's thermal signal over distances comparable to its own dimensions. By setting $L = D$, we can obtain the time scale $\tau_D \sim 1$ yr for diffusion of this signal through the convection zone. But our modeling of the isotherm changes and of the time variation of the total heat flux through the photosphere shows clearly that these diffusive signal times are of limited relevance to the time scale of storage of the blocked heat. The much longer time scale required for the convection zone to return to thermal equilibrium after the spot is formed is determined mainly by the radiative boundary condition imposed on equation (1), rather than by the diffusive process described by the equation itself.

Our finding that small thermal obstacles placed at the photosphere are able to cause efficient storage of the blocked flux is in good agreement with the independent analysis of this problem by Spruit (1981; 1982*a*, *b*). Spruit's study is simplified in many respects to allow analytical treatment. But his main conclusions coincide well with those found in our two-dimensional numerical integrations and provide useful complementary insights. For instance, in our model, the inevitable storage of energy in the expansion of the heated hydrostatic convection zone is not considered. From the virial theorem, we would expect roughly two-thirds of the stored energy to appear as an increase of potential energy of the convective envelope, compared to the one-third stored in thermal energy, considered in our model. Spruit's integration over a polytropic envelope allows a direct estimate of the (negligible) increase of photospheric radius outside the spot that would accompany this storage.

The thermal effect of magnetic blocking of solar heat flow has also been estimated by Dearborn and Blake (1982). Their model also arrives at a long storage time of order τ_R, but that result is less surprising, since they assume perfect redistribution of the locally blocked heat, rather than demonstrating that this redistribution actually occurs in a plausible convection zone model.

Since the efficient storage found in all these studies arises mainly from the high thermal inertia of the convection zone, the main results are relatively insensitive to uncertainties in the strict applicability of the diffusion approximation to convection around the spot, or to the constraints of $\rho(z)$, $C_p(z)$, and $K(z)$ invariant in time, that we have used in our model. The time scale τ_R does depend somewhat on the diffusion rate in our model through the scale l in equation (6). Specifically, if our estimates of the tensor eddy conductivity were too high, only local surface layers might be affected by the spot, leading to $l \ll D$. The bright ring observations of Fowler, Foukal, and Duvall (1982) indicate that the surface value of K or its gradient with depth may be lower than the values used in our model. Another source of error lies in the uncertainty of the convection zone depth D. But the basic conclusion that storage will be very effective over time scales of the observed irradiance dips, or even the 11 year cycle, is difficult to avoid, unless mixing length models of heat transport in the convection zone are seriously wrong.

Parker (1974) has suggested that spot coolness might result from conversion of convective energy to Alfvén waves, which would then escape from the spot, mainly into the convection zone (Thomas 1978). The attractiveness of this idea is somewhat diminished by Spruit's (1977) calculations showing that Biermann's (1941) mechanism of simple convective blocking is not inconsistent with observations of only rather weak bright rings around spots (see also Sweet 1955). Still, substantial contribution from Alfvén-wave cooling cannot be ruled out. It is clear that if such were the case, the fraction of energy redirected downward in the form of waves could not be modelled in the way described here.

But even in this case, relatively long storage times (compared to the 10–20 days of the irradiance dips) might be expected, so that equation (5) might still hold. However, the conclusion that spots should produce modulations over the solar cycle would no longer be secure.

In summary, the model described here seems to provide a rather simple explanation of the observed amplitude, duration, shape, and phase of the sunspot-correlated decreases of solar irradiance. The storage mechanism we have identified places no special requirements on a physical relationship between spots and faculae. In fact thermal blocking might also operate for facular flux tubes, since recent photometric observations (Foukal, Duvall, and Gillespie 1982) show that faculae are dark in the continuum, when observed near Sun center.

The excess brightness of faculae near the limb (e.g., Richardson 1933; Chapman 1977) is well explained as a result of viewing the hot internal wall of a slender magnetohydrostatic flux tube (Spruit 1976; Foukal, Duvall, and Gillespie 1982). It certainly does not require any detailed energy balance between spots and faculae, although it probably does produce a modulation of the irradiance data (Foukal, Mack, and Vernazza 1977; Foukal and Vernazza 1979; Hudson and Willson 1981; Oster, Schatten, and Sofia 1982). This modulation may well also indicate a facular contribution to $L_\odot$ due in part to the hot wall effect and in part to the blocking. An additional contribution to changes in $L_\odot$ is likely to come from the enhanced ultraviolet flux from the non-thermally heated facular chromosphere (Heath 1980). These considerations certainly suggest that other contributions to the 11 year modulation of $L_\odot$ are to be expected, but we see no evidence suggesting that the spot and facular contributions to $L_\odot$ exactly cancel over that time scale, or any other.

This work was supported at AER, Inc. under NSF grant ATM-8004346.

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P. FOUKAL, L. A. FOWLER, and M. LIVSHITS: Atmospheric and Environmental Research, Inc., 840 Memorial Drive, Cambridge, MA 02139